

Precalc BC Parametric Equations.

1. A particle P starts at $(0,1)$ and moves 2 units to the right and 3 units up every second along a line. Find a pair of parametric equations to describe the position of P after t seconds.

$$x(t) = 2t$$

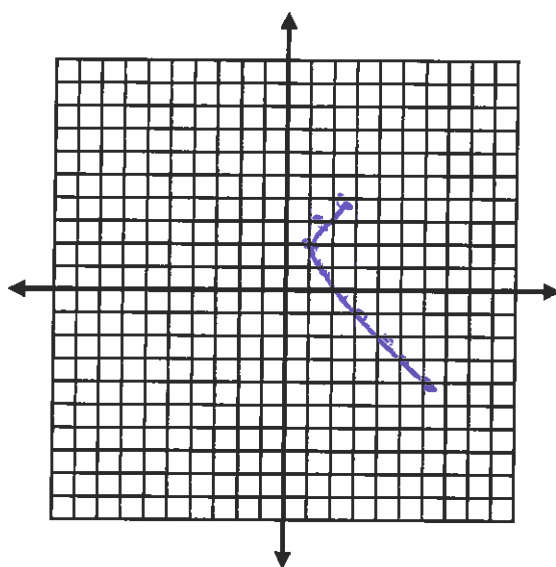
$$y(t) = 3t + 1$$

2. Fill in the table and sketch the parametric equation for $-2 \leq t \leq 6$

$$x = \sqrt{t^2 + 1}$$

$$y = 2 - t$$

Fx1
This is a
hyperbolic
segment.



t	x	y
-2	2.2	4
-1	1.4	3
0	1	2
1	1.4	1
2	2.2	0
3	3.2	-1
4	4.5	-2
5	5.1	-3
6	6.1	-4

3. For the parametric equations $x = t$ and $y = t^2$

a) Use Desmos or Geogebra to graph these equations.

b) Graph $x = t - 1$ and $y = t^2$.

How does this compare to the graph in part (a)?

Translate left 1 unit

c) Graph $x = t$ and $y = t^2 - 3$.

How does this compare to the graph in part (a)?

Translate down 3 units

d) Write parametric equations which will give the graph in part (a) a vertical stretch by a factor of 2 and move the graph 5 units to the right.

$$x = t + 5, \quad y = 2t^2$$

4. For the parametric equations $x = t$ and $y = |t|$

a) Use Desmos or Geogebra to graph.

b) Graph $x = t + 2$ and $y = |t| - 1$.

How does this compare to the graph in part (a)?

Translate $\langle 2, -1 \rangle$

c) Write a pair of parametric equations that will move the graph in part (a) 4 units to the left and three units down.

$$x = t - 4, \quad y = |t| - 3$$

d) Describe how the numbers $\frac{1}{2}$, 1 and 3 in the following parametric equations transform the graph from part (a). $x = \frac{1}{2}t + 1$ and $y = |t| + 3$

horiz compression \leftarrow translate right 1 unit \rightarrow translate up 3

5. Write a new set of parametric equations with the following transformations for $x = t^4$ and $y = 2t$

a) Shift right 7 and stretch vertically by a factor of 3.

$$x = t^4 + 7, \quad y = 6t$$

b) Shift up 7 and stretch horizontally by factor of 4.

$$x = 4t, \quad y = 2t + 7$$

c) Dilate in both directions by a factor of 4, shift left 3, and down 5.

$$x = 4t^4 - 3, \quad y = 8t - 5$$

6. Two tankers leave Corpus Christi at the same time traveling toward St. Petersburg, which is 900 miles east of Corpus Christi. Tanker A travels at 18mph and Tanker B travels at 22mph.

a) Write parametric equations for the situation.

$$x_A = 18t, \quad x_B = 22t$$

b) How long does it take the faster tanker to reach St. Petersburg?

$$x_B(t) = 900 \rightarrow t = 40.90 \text{ hrs}$$

c) Where is the slower tanker when the faster tanker reaches its destination?

$$x_A(40.90) = 736 \text{ mi}$$

d) When, during the trip, is the faster tanker exactly 82 miles in front of the slower tanker?

$$x_B - x_A = 82 \rightarrow t = 20.5 \text{ hrs}$$

e) During what part of the trip are the tankers less than 60 miles apart?

$$x_B - x_A < 60 \\ \rightarrow t < 15 \text{ hrs}$$

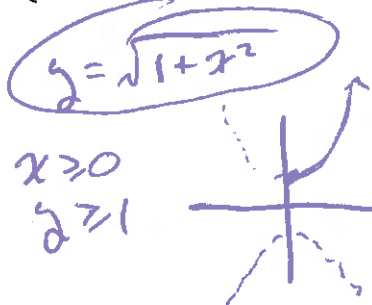
(Also after 46.7 hrs assuming Tanker B stays in port)

7. For each parametric equation of a curve given below...

- Identify the graph by eliminating the parameter.
- Sketch the graph. (Be sure to respect domain and range restrictions.)

a) $\begin{cases} x = \sqrt{t} \rightarrow t = x^2 \\ y = \sqrt{1+t} \end{cases}$

This is one branch of a hyperbola

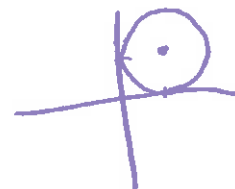


b) $\begin{cases} x = 1 + \cos t \rightarrow x-1 = \cos t \\ y = 1 - \sin t \rightarrow y-1 = -\sin t \end{cases}$

$(x-1)^2 + (y-1)^2 = 1$

$0 \leq x \leq 2$

$0 \leq y \leq 2$

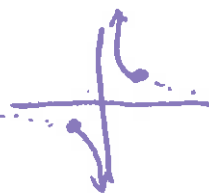


c) $\begin{cases} x = \cos t \\ y = \sec t \end{cases}$

$y = \frac{1}{x}$

$x \in [-1, 0) \cup (0, 1]$

$y \in (-\infty, -1] \cup [1, \infty)$

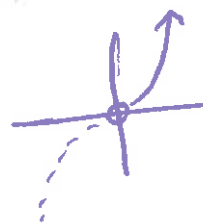


d) $\begin{cases} x = 2^t \\ y = 8^t \end{cases}$

$y = x^3$

$x > 0$

$y > 0$



e) $\begin{cases} x = \cos t \\ y = 3 \sin^2 t \end{cases}$

$\frac{y}{3} = \sin^2 t$

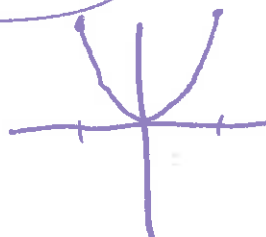
$x^2 + \left(\frac{y}{3}\right) = 1$

$y = -3x^2$

$-1 \leq x \leq 1$

$0 \leq y \leq 3$

parabolic segment



f) $\begin{cases} x = 4 \sec t \\ y = 3 \tan t \end{cases}$

$\frac{x}{4} = \sec t$

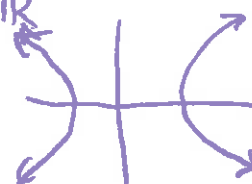
$\frac{y}{3} = \tan t$

Hyperbola $\left(\frac{y}{3}\right)^2 + 1 = \left(\frac{x}{4}\right)^2$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

$x < -4$ or $x > 4$

$y \in \mathbb{R}$



8. Do the following sets of parametric equations cross? If so, do they "collide"? Justify your answer. If they collide state the time and place. (Start by eliminating the parameter, solve the system of equations to find if/where the paths cross. Then solve for t).

a) $x_1 = 4t$ and $x_2 = 5t - 6$
 $y_1 = \frac{1}{2}t + 5$ $y_2 = t + 2$

$$x = 8y - 40 \quad x = 5(y - 2) - 6$$



$$y = 8$$

$$x = 24$$

$$(24, 8)$$

paths cross

$$x_1 = 24 \rightarrow t = 6$$

$$x_2 = 24 \rightarrow t = 6$$

same place, same time
points collide

b) $x_1 = 3 - t$ and $x_2 = 3 - 2t$
 $y_1 = 2t + 1$ $y_2 = 2 + 3t$

$$y = 2(3 - x) + 1 \quad y = 2 + 3\left(\frac{3 - x}{2}\right)$$



$$x = 1$$

$$y = 5$$

paths cross

$$x_1 = 1 \rightarrow t = 2$$

$$x_2 = 1 \rightarrow t = 1$$

points do not collide

c) $x_1 = 3 - t$ and $x_2 = t + 19$
 $y_1 = t^2 - 60$ $y_2 = t + 12$

$$y = (3 - x)^2 - 60 \quad y = (x - 19) + 12$$

$$x^2 - 7x - 44 = 0$$

$$x = +11 \text{ or } -4$$

$$(11, 4)$$

$$(-4, -11)$$

Paths
cross
twice

$$x_1 = 11 \rightarrow t = -8$$

$$x_2 = 11 \rightarrow t = -8$$

collide!

$$x_1 = -4 \rightarrow t = 7$$

$$x_2 = -4 \rightarrow t = -23$$

miss!

9. Horizontal Motion of a Particle.

a) Graph the parametric equations on your calculator with the provided

window: $x = 4t^3 - 16t^2 + 15t$; $y = 2$

T [0, 5] step 0.05 X [-4, 6] scl 1 Y [-3, 6] scl 1

b) When and where does the particle reverse direction? (Trick question – it will be hard to tell)

It is easier to answer this in step (c) below

c) Modify the parametric equations from part (a): $x = 4t^3 - 16t^2 + 15t$, $y = t$
Describe how this changes your "view."

The graph is 2-D now, y-axis = t

d) Revisit question (b). Use *trace* on your calculator to help you.

Reverses @ $t \approx 0.7$ & $t = 2$

10. Mary and Kelly are standing 78 feet apart. At the same time, they each throw a softball toward each other. Mary throws her softball with an initial velocity of 45 ft/sec with an angle of elevation of 44° . Kelly throws her softball with an initial velocity of 41 ft/sec and an angle of elevation of 39° .

a) Write the sets of parametric equations to simulate the motion of the softballs.

$$x_m = (45 \cos 44^\circ)t$$

$$y_m = -16t^2 + (45 \sin 44^\circ)t$$

$$x_k = 78 - (41 \cos 39^\circ)t$$

$$y_k = -16t^2 + (41 \sin 39^\circ)t$$

Use your calculator, Desmos or geogebra to answer the following questions - not algebra.

b) Will the softballs collide? Justify your answer.

No, Kelly's is always lower

c) Which softball hits the ground first?

Kelly's @ about 1.61 sec

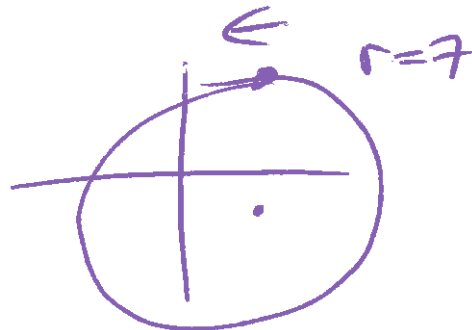
d) How far does each softball travel in the horizontal direction?

Kelly \approx 52 ft, Mary \approx 63 ft

11. Find a pair of parametric equations to describe the position P of a particle moving in a counterclockwise direction around a circle of radius 7 with center at $(3, -1)$, starting at the 12:00 position.

$$x = -7 \sin t + 3$$

$$y = 7 \cos t - 1$$



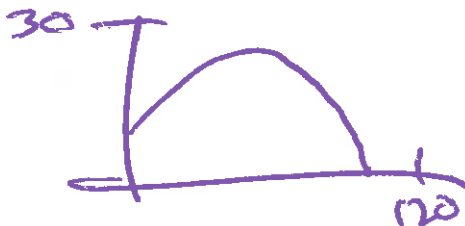
12. A dart is thrown from a point 5 feet above the ground with an initial velocity of 58 ft/sec and angle of elevation of 41° . Assume the only force acting on the dart is gravity.

- a) Write a pair of parametric equations to simulate the motion of the dart.

$$x(t) = (58 \cos 41^\circ)t$$

$$y(t) = -16t^2 + (58 \sin 41^\circ)t + 5$$

- b) Sketch a graph of the motion and give an appropriate window.



- c) What is the maximum height reached by the dart?

$$\approx 27.62 \text{ ft}$$

- d) When and where will the dart hit the ground?

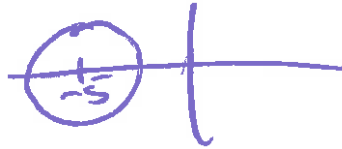
$$\approx 109 \text{ ft}$$

13. For the curve $(x + 5)^2 + y^2 = 4$ complete the following:

a) Write a pair of parametric equations for the curve.

$$x = 2 \cos t - 5 \quad y = 2 \sin t$$

b) Give an appropriate window and sketch the graph.



c) How could you graph just the top half of the curve?

$$0 \leq t \leq \pi$$

d) How could you graph just the left side of the curve?

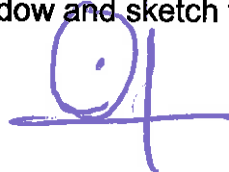
$$\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

14. For the curve $\frac{(x+3)^2}{1} + \frac{(y-4)^2}{16} = 1$ complete the following:

a) Write a pair of parametric equations for the curve.

$$x = \cos t - 3 \quad y = 4 \sin t + 4$$

b) Give an appropriate window and sketch the graph.



c) How could you graph just the right side of the curve?

$$-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

d) How could you graph just the bottom half of the curve?

$$\pi \leq t \leq 2\pi$$

15. Write an equation for the cycloid described by a wheel with radius 15 cm. Check your answer on Desmos or Geogebra.

$$\begin{aligned}x &= 15t - 15 \sin t \\y &= 15 - 15 \cos t\end{aligned}$$

16. Modify your equation from # 15 for each of the following (non-cumulative).

- a) Have the wheel traveling to the *left* and starting at the point $(30\pi, 0)$.

$$\begin{aligned}x &= -15t + 15 \sin t + 30\pi \\y &= 15 - 15 \cos t\end{aligned}$$

- b) Have your wheel travel twice as fast.

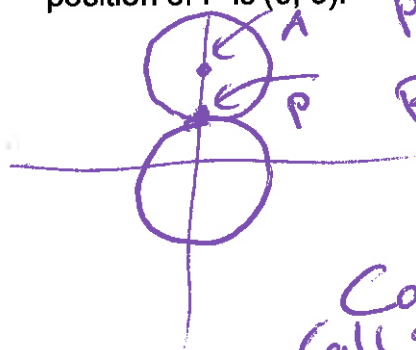
$$\begin{aligned}x &= 30t - 15 \sin 2t \\y &= 15 - 15 \cos 2t\end{aligned}$$

- c) Have your wheel travel up a ramp that makes an angle of 20° with the x-axis. (Hint: Write a parametric equation for that line, use a right triangle whose hypotenuse is the linear velocity of the wheel).

ramp: $x = 15t, y = 7.5t$

cycloid: $x = 15t - 15 \sin t, y = 15 + 7.5t - 15 \cos t$

17. A circle of radius 5 is rotating clockwise around a second circle with the same radius, centered at the origin. Write an equation of the *cardioid* traced by a point P on the circumference of the first circle. Assume the initial position of P is $(0, 5)$.



Path A $x = 10 \sin t, y = 10 \cos t$

Path P (relative to A)

$$x = -5 \sin 2t, y = -5 \cos 2t$$

Cardioid
(add 2 paths)

$$\begin{aligned}x &= 10 \sin t - 5 \sin 2t \\y &= 10 \cos t - 5 \cos 2t\end{aligned}$$