Pre Calc. BC   Abstract Algebra - REVIEW   **Name___________**

1. Consider the relation $(\mathbb{Q}^*, *)$, where $a * b = \frac{a}{b}$ and $\mathbb{Q}^*$ is rational numbers without zero. Discuss the truth of the following statement:

"*Each element is its own inverse for this operation.*"

2. Determine which (if any) relations below form abelian groups. Demonstrate **all** the properties. Be clear, concise, and complete.

a) $(\mathbb{Z}_9^*, \otimes)$

b) $(W, *)$ *where* $x * y = |x - y|$

c) $\circ$ *as shown in the table at right*

$\circ$	!	#	$	%
!	!	%	!	#
#	%	#	#	$
$	!	#	$	%
%	#	$	%	!

3. Consider the operation $*$, such that $x * y = \frac{xy}{2}$.

a) For which of the following sets is $*$ closed?

i) $\mathbb{Z}$ ii) $\mathbb{Q}^+$ iii) $\mathbb{R}$ iv) {even integers} v) {0, 1, 2}

b) Is the operation associative? Demonstrate it.

c) Is there an identity? Find it.

d) What is the inverse of 5?

e) Is ({2, 4, 6, 8, ...}, $*$) an abelian group? Explain.

4. Let H be the set of symmetry *rotations* for a regular octagon.

a) Make a list of the elements of H. Indicate the order of each element.

b) Is (H, $*$) a group?

c) List all the subgroups of H.

d) Is (H, $*$) isomorphic to $sym(\square)$ Be complete and specific in your answer.

5. Five groups are defined below. Determine which are isomorphic.

a) ($\mathbb{Z}_4$, $\oplus$)

b) ($\mathbb{Z}^*_5$, $\otimes$)

c)

ø	**a**	**b**	**c**	**d**
a	b	a	d	c
b	a	b	c	d
c	d	c	b	a
d	c	d	a	b

d) The symmetry group for a rhombus.

e) The *rotational* symmetry group of a square.

6. List all possible subgroups for ($\mathbb{Z}_{12}$, $\oplus$).

7. Consider the two statements below and provide a reason, example or counter example

a) *A non-cyclic group can have a cyclic sub-group.*

b) *An non-abelian group can have an abelian sub-group.*

8. Consider the group ($\mathbb{Z}^*_7$, $\otimes$)

a) List all the elements with their orders.

b) What is the inverse of 4?

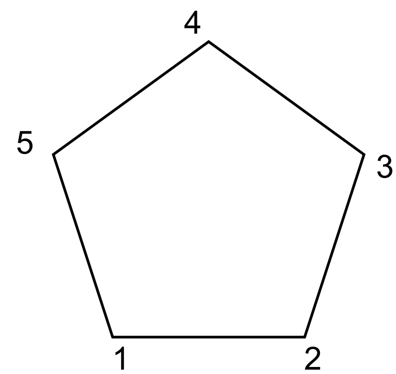
c) List a generator of $\mathbb{Z}^*_7$.

d) Find a subgroup of order 3.

e) Explain why ($\mathbb{Z}_7$, $\oplus$) has no subgroups. *Hint: what is the order of* $\mathbb{Z}_7$?

9. Consider the symmetry group, H, for the regular pentagon shown at right.

a) Draw or describe one transformation that is an element of H. (*NOT the identity).*



b) What is the order of H?

c) Compare H to $(\mathbb{Z}_{10}, \oplus)$. Explain why these two groups are not isomorphic.

d) Find two isomorphic subgroups from each of these groups.