Precalc BC Series, Inductive Proofs and e Extra Review

1. Given a geometric series starting with $a_n = 1$, find the range of values to which the series can converge.

$S_n = \frac{1}{1-r}$ if $0 < r < 1 \rightarrow S_n > 1$

if $-1 < r < 0 \rightarrow \frac{1}{2} < S_n < 1$

therefore $S_n > \frac{1}{2}$ but $S_n \neq 1$

2. The first and third terms of a sequence are, respectively, ¾ and ½ . Find the difference between the arithmetic mean and the harmonic mean of these two terms.

$A.M = \frac{\frac{3}{4}+\frac{1}{2}}{2} = \frac{\frac{5}{4}}{2} = \frac{5}{8}$, $HM \; \frac{2\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)}{\frac{3}{4}+\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$, $\frac{5}{8} - \frac{3}{5} = \boxed{\frac{1}{40}}$

3. Find the sum of the geometric series: 17 – 51 + 153 - …+ 1,003,883.

$r = -3$, $n = \left(\log_3 \frac{1{,}003{,}883}{17}\right) + 1 = 11$, $S_n = 17\,\frac{(-3)^{11}-1}{-3-1} = \boxed{752{,}879}$

4. Evaluate: a) $\sum_{n=2}^{20} 5^n$ b) $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ $\boxed{= \frac{1}{e^2}}$

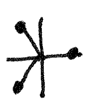
$\underbrace{25 + 125 + \ldots + 5^{20}}_{19 \text{ terms}}$ $S_n = 25\,\frac{5^{19}-1}{5-1} = \boxed{1.192 \times 10^{14}}$

5. Given a geometric series with r a non-real number, $a_1 = a_4 = 2$, find $a_{20} \times a_{21}$.

$r = \sqrt[3]{1} = \sqrt[3]{1 \operatorname{cis} 0} = 1 \operatorname{cis} \pm 120°$ $a_{20} \cdot a_{21} = 4 \cdot 1 \operatorname{cis}(120 \times 39)$

$a_{20} = 2(1 \operatorname{cis} 120°)^{19}$, $a_{21} = 2(1 \operatorname{cis} 120°)^{20}$ $\boxed{= 4}$

Think geometrically!



$r^{39} = (r^3)^{13} = 1^{13} = 1$

6. An arithmetic series has $a_7 = 35$ and $a_{19} = -1$. Find the sum of the first 20 terms.

$d = \frac{-1-35}{19-7} = \frac{-36}{12} = -3$

$a_1 = 35 - (6)(-3)$

$a_1 = 53$

$a_{20} = -4$

$S_{20} = (53 + -4)\frac{20}{2} = \boxed{490}$

7. The coefficient of the x^3y^4 term of the expansion of $(2x+ky)^7$ is 672,280. Find the value of k.

$${}_7C_3(2)^3(k)^4 = 672,280 \rightarrow k^4 = 2401$$

$$k = \pm 7$$

8. Write the first five terms in the expansion of cos(π).

$$\frac{\pi^0}{0!} - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} = -.976 \approx -1$$

9. Prove inductively: $1-2+4-8+\ldots(-2)^{n-1} = \dfrac{1-(-2)^n}{3}$

anchor

$k=1$

$$(-2)^{1-1} \stackrel{?}{=} \frac{1-(-2)^1}{3}$$

$$1 = 1 \checkmark$$

Inductive Hypothesis

$f(k)=g(k) \Rightarrow f(k+1)=g(k+1)$

$$f(k+1) = 1-2+4-8+\ldots(-2)^{k-1} + (-2)^{(k+1)-1}$$

$$= \frac{1-(-2)^k}{3} + (-2)^k$$

$$= \frac{1-(-2)^k}{3} + \frac{3(-2)^k}{3}$$

$$= \frac{1+2(-2)^k}{3}$$

$$= \frac{1-(-2)(-2)^k}{3}$$

$$= \frac{1-(-2)^{k+1}}{3} = g(k+1)$$

$\therefore$ true for all $n \in \mathbb{N}$