PreCalc BC Review Name: Key

1. Write an equation for the transformation of $y=\sqrt{1-x^2}$ shown below.

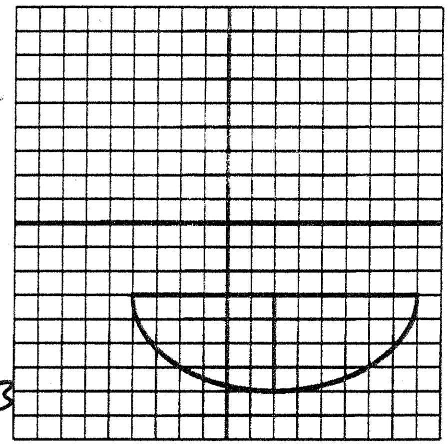
Look at center for translations.

$T_{(2,0)}$ & $T_{(0,-3)}$

Look at radii for dilations

$D_V = 4$, $D_H = 6$

Also $r_{x\text{-axis}}$



$\rightarrow -4f\left(\frac{x-2}{6}\right)-3 \rightarrow y=-4\sqrt{1-\left(\frac{x-2}{6}\right)^2}-3$

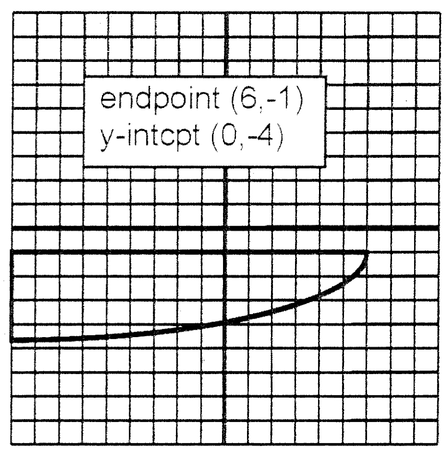
2. Write an equation for the transformation of $y=\sqrt{x}$ shown below.

$T_{(6,0)}$ & $T_{(0,-1)}$

r_x & r_y

There are several ways to determine dilation, but pretend the y-intcpt used to be (1,1)

hence $D_H = 6$, $D_V = 3$



$\rightarrow -3f\left[\left(\frac{x-6}{6}\right)\right]-1 \rightarrow -3\sqrt{\frac{6-x}{6}}-1$

3. Given the functions $f(x)=\dfrac{1}{x+3}$ and $g(x)=1-x^2$

a. Find the domain of *f(g(x))*. $\rightarrow = \dfrac{1}{1-x^2+3} = \dfrac{1}{4-x^2}$ D: $x \neq \pm 2$

b. Find the inverse of *f(x)*. $x=\dfrac{1}{y+3} \rightarrow y=\dfrac{1-3x}{x}$

c. Describe *f* and *g* as transformations of basic functions. f: $T_{(-3,0)}$

d. Find *x* such that *g(f(x))* = -3 g: $T_{(0,1)}$ & r_x

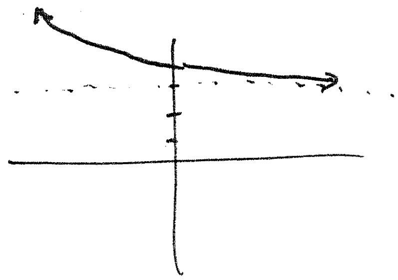
$g(f(x)) = 1-\dfrac{1}{(x+3)^2} = -3$

$\rightarrow 4 = \dfrac{1}{(x+3)^2} \rightarrow x+3 = \pm\frac{1}{2} \rightarrow x = -\frac{5}{2}, -\frac{7}{2}$

4. Sketch each of the functions below. Then check with your calculator.

a. $y = 2^{-x} + 3$

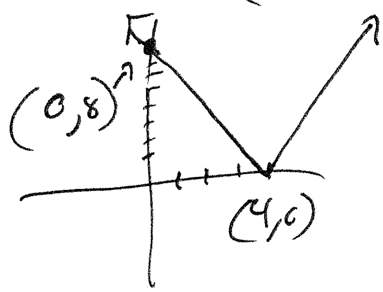
r_y & $T(0,3)$



b. $y = 2|4 - x|$

$= 2|x-4|$

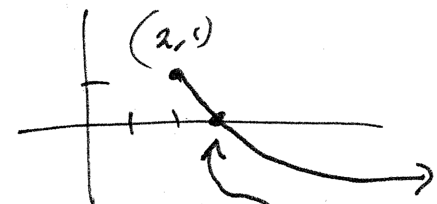
$D_v = 2,\ T(4,0)$



c. $y = 1 - \sqrt{2x-4} = 1 - \sqrt{2(x-2)}$

$T(0,1)$ r_y

$T(2,0)$ $D_v = \frac{1}{2}$



$1 - \sqrt{2x-4} = 0$

$\rightarrow x = 5/2$

5. Find the following limits:

a. $\lim\limits_{x \to 3} \dfrac{x-3}{3}$

$\dfrac{0}{3}$

$\boxed{=0}$

b. $\lim\limits_{x \to -1} \dfrac{x+1}{2x+2}$

$\dfrac{0}{0}$

$\dfrac{\cancel{(x+1)}}{2\cancel{(x+1)}}$

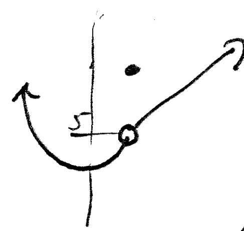
$\boxed{=\frac{1}{2}}$

c. $\lim\limits_{x \to \infty} \dfrac{4x+1}{2x+2}$

$\dfrac{4x}{2x}$

$\boxed{=2}$

d. $\lim\limits_{x \to 3} \begin{cases} x^2 - 4, & x < 3 \\ 7, & x = 3 \\ x + 2, & x > 3 \end{cases}$



lim = 5 (not 7)

6. Chose a function from your "library" that satisfies the following conditions -- no fair looking at your library while you do this.

a) Bounded below, but no extrema $y = e^x$

b) Odd symmetry, restricted domain $y = \dfrac{1}{x}$

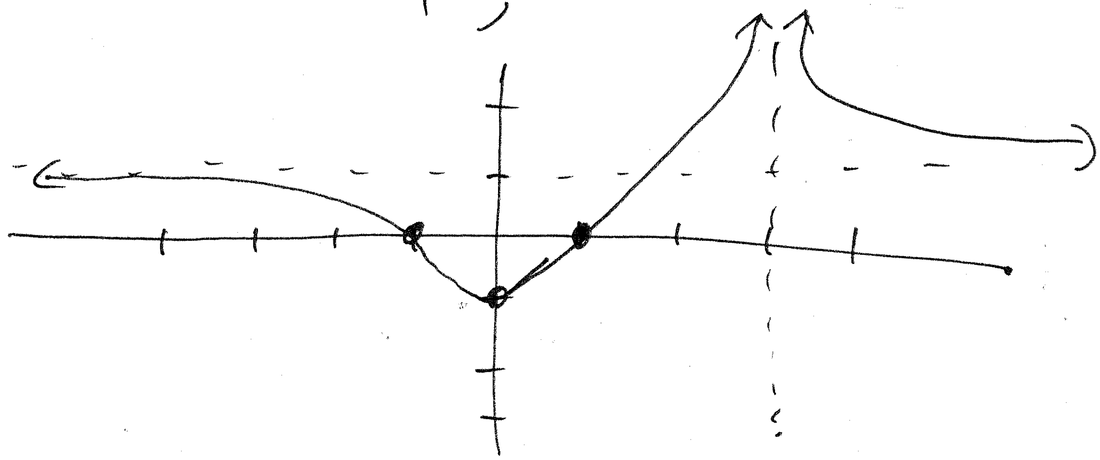
c) Continuous, odd symmetry, restricted range $y = \sin x$

7. Analyze the following function, then sketch: $y = \dfrac{x^2-1}{(x-3)^2}$

$\lim_{x \to \infty} = \dfrac{x^2}{x^2} = 1$ → horiz asymp: $y = 1$

Domain: $x \neq 3$, $\lim_{x \to 3} = \dfrac{8}{0}$ → vert asympt (even)

y-intcpt $f(0) = \dfrac{-1}{9}$, x intcpt → $x = \pm 1$



8.

a. What is the difference between an essential versus removable discontinuity?

lim DNE v. lim exists

b. What is a one to one function? Can a function be its own inverse? How are these two questions related?

must pass horiz & vert line test, 1-1, yes (ex $1/x$)

if function is 1-1 & reflects over $y = x$

c. Can a function have two horizontal asymptotes? More than two?

D f(g) is a subset of D g

d. What is the relationship between the domain of *f(g)* and the domains of the two functions *f* and *g*?

Two horiz asympt is max

e. Which of the following attributes are always unchanged by a vertical dilation?

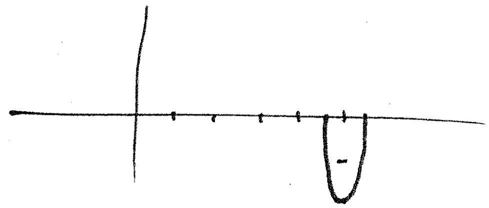
x-intcpts, y-intcpt, symmetry, domain, range, vertical asymptotes

f. Same as previous question: what about for a reflection over the y-axis?

y-intcpt, sym, range

Extra Problems

2) $f(x) = \sqrt{1-x^2}$, sketch $-2f(3x-15) = -2f[3(x-5)]$

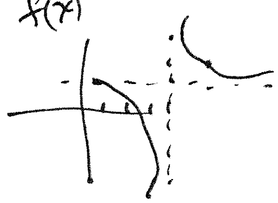


③ $f(x) = \dfrac{x}{x-4}$

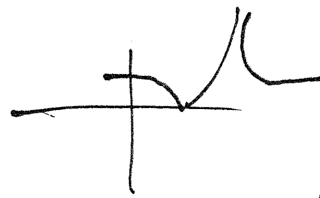
$f^{-1} = \dfrac{4x}{x-1}$ D: $x \neq 1$

R: $y \neq 4$ (from $f(x)$)

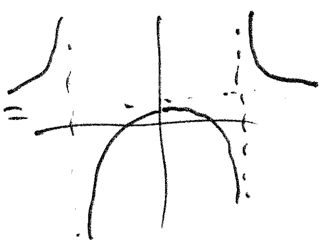
$f(x)$



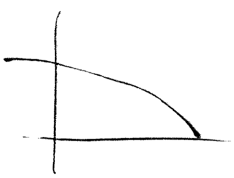
$|f(x)| =$

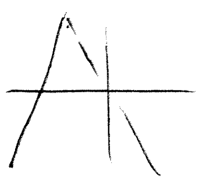


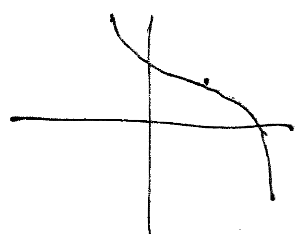
$f(|x|) =$



4.







$y = \dfrac{\sqrt{4-x}}{2}$

$y = 8 - \left|\dfrac{x+3}{\left(\frac{8}{3}\right)}\right|$

$y = 16 - 2(x-4)^3$

$\cancel{y = 16 - \frac{3}{4}(x-4)^3}$

5. $\lim\limits_{x \to \infty} f^{-1} = +\infty$

$f(x) = \ln(3-x)$

$f'(x) = 3 - e^x$