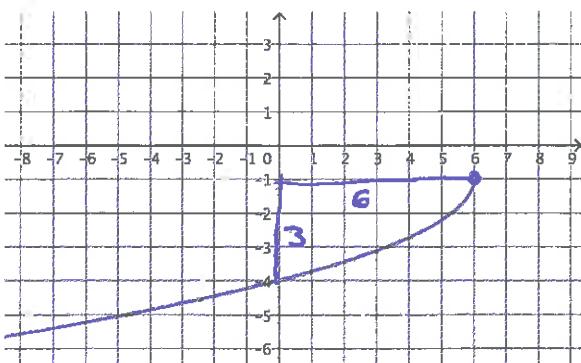


1. Write an equation for each transformation of a basic function shown below.



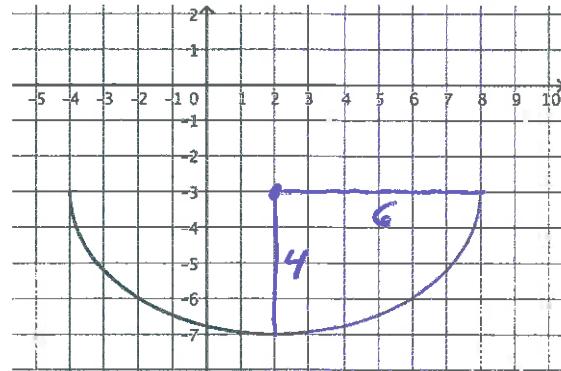
$T(6, -1)$ - based on origin

$\cap x\text{-axis and } \cap y\text{-axis}$

Dilation: There are a few ways to do this based on the triangle above. I see h. compression b/c $\Delta y = 3 \rightarrow \Delta x = 9$ for \sqrt{x} function \therefore compress by $\frac{1}{3}$

So we want $-f\left[-\frac{3}{2}(x-h)\right] - 1$

$$\rightarrow y = -\sqrt{-\frac{3}{2}(x-6)} - 1$$



$T(2, -3)$ - based on origin

$\cap x\text{-axis}$

Vertical stretch $\times 4$

Horiz stretch $\times 6$

So we want $-4f\left(\frac{x-2}{6}\right) - 3$

$$\rightarrow y = -4\sqrt{1 - \left(\frac{x-2}{6}\right)^2} - 3$$

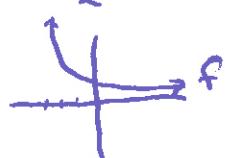
2. Given the functions $f(x) = \frac{1}{\sqrt{x+3}}$ and $g(x) = 1-x^2$

- a) Find the domain of $f(g(x))$.

$$f(g) = \frac{1}{\sqrt{4-x^2}}, \text{ so } 4-x^2 > 0 \rightarrow (2+x)(2-x) > 0 \rightarrow \begin{array}{c|ccc|c} & + & + & + & - \\ \hline -2 & & & & \\ 2 & & & & \end{array}$$

- b) Find the inverse of $f(x)$ with appropriate domain.

$$x = \frac{1}{\sqrt{y+3}} \rightarrow y+3 = \frac{1}{x^2} \rightarrow y = \frac{1}{x^2} - 3 \text{ is } f^{-1}$$



Range of $f(x)$ is $y > 0$, \therefore Domain of f^{-1} is $x > 0$

- c) Describe $g(f(x))$ as a transformation of a basic function.

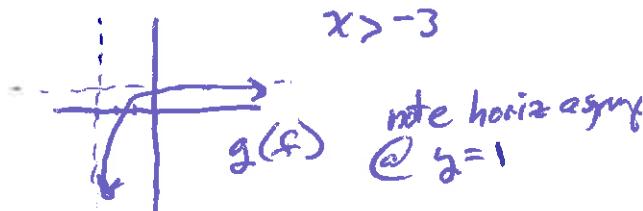
$$g(f) = 1 - \frac{1}{x+3}, \text{ so if } h(x) = \frac{1}{x}, \text{ this is } 1 - f(x+3)$$

However there is a domain restriction:

- d) Find x such that $g(f(x)) = 1$

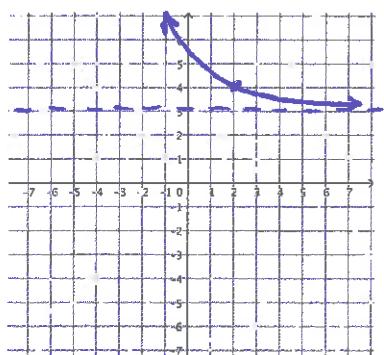
$$1 - \frac{1}{x+3} = 1$$

$$\rightarrow \frac{1}{x+3} = 0 \quad \text{impossible!}$$

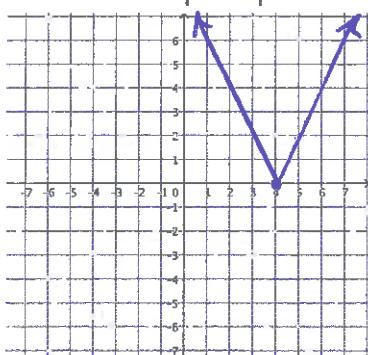


3. Sketch each of the functions below. Then check with your calculator.

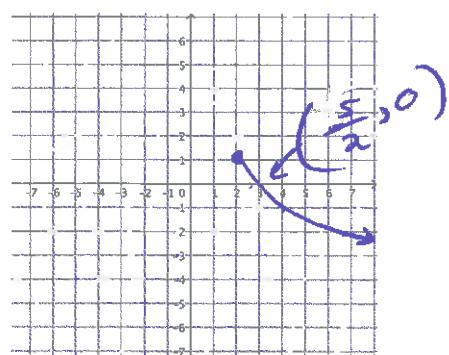
a. $y = e^{2-x} + 3$



b. $y = 2|4-x|$



c. $y = 1 - \sqrt{2x-4}$



This is $f[-(x-2)] + 3$

so $T\langle 2, 3 \rangle$, Γ_y -axis

Pay attention to
horiz asympt &
y-intcpt @ (0,1)

This is $2f[-(x-4)]$

so vert stretch
 x^2

$T\langle 4, 0 \rangle$

and Γ_y -axis, which
is irrelevant since
even function

This is $-f[2(x-2)] + 1$

Γ_x -axis

H. compr. $\times \frac{1}{2}$

$T\langle 2, 1 \rangle$

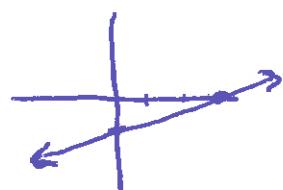
start w/ origin,
solve for x-intcpt to
show dilation.

4. Find the following limits:

a. $\lim_{x \rightarrow 3} \frac{x-3}{3}$

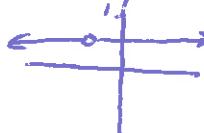
$$= \frac{0}{3}$$

$\boxed{= 0}$



b. $\lim_{x \rightarrow 1} \frac{x+1}{2x+2}$

$$= \frac{0}{0}, \text{ so factor: }$$

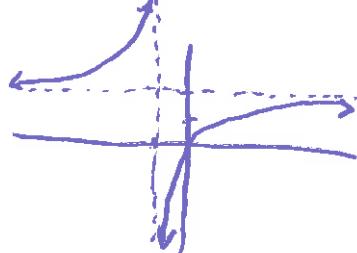


$$\frac{(x+1)}{2(x+1)} = \boxed{\frac{1}{2}}$$

c. $\lim_{x \rightarrow \infty} \frac{4x+1}{2x+2} = \frac{\infty}{\infty}$

so obliterate!

$$\frac{4x}{2x} = 2$$

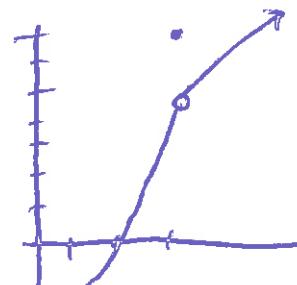


d. $\lim_{x \rightarrow 3} \begin{cases} x^2 - 4, & x < 3 \\ 7, & x = 3 \\ x+2, & x > 3 \end{cases}$

$$(3)^2 - 4 = 5$$

$$(3) + 2 = 5$$

so $\lim_{x \rightarrow 3^+} = \lim_{x \rightarrow 3^-} = \boxed{5}$



5. Choose a function from your "library" that satisfies the following conditions -- no fair looking at your library while you do this.

a) Bounded below, but no extrema

$$y = e^x \quad \cancel{\text{graph}}$$

$$\text{or } y = \frac{1}{1+e^{-x}}$$



b) Odd symmetry, restricted domain

$$y = \frac{1}{x} \quad \cancel{\text{graph}}$$

c) Continuous, odd symmetry, restricted range

$$y = \sin x \quad \cancel{\text{graph}}$$

6. Analyze the following function, then sketch: $y = \frac{x^2-1}{(x-3)^2} = \frac{x^2-1}{x^2-6x+9} = \frac{(x+1)(x-1)}{(x-3)^2}$

Domain: $x \neq 3$

$$\lim_{x \rightarrow 3} = \frac{8}{0} \rightarrow \text{v. asympt}$$

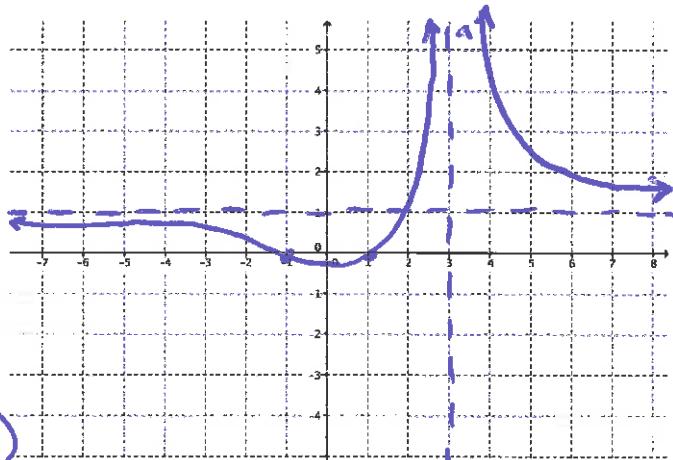
$(x-3)^2 \rightarrow \text{even asympt}$

$$y\text{-intcpt } (0, -\frac{1}{9})$$

$$x\text{-intcpt } x^2-1=0 \rightarrow (\pm 1, 0)$$

$$\lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1$$

end behavior: $y=1$

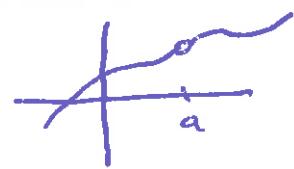


7.

- a) Can a function that has a discontinuity still have a limit that exists for each value of x in the domain?

This is the case for any function with removable discontinuities

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$$



- b) Must a continuous function with odd symmetry pass through the origin?

Yes, if continuous on \mathbb{R} . Otherwise if $f(0) = a$ then $f(0)$ also $= -a$ (R_{180°) \rightarrow not a function

- c) What is the symmetry of a composition of an even and an odd function?

Does the order in which they are composed matter?

Either way the result is an even function.

$$\begin{aligned} f(g(x)) &= \text{even}, \quad g(x) = \text{odd} \\ f(g(-x)) &= f(-g(x)) = f(g(x)) \quad \text{is even} \\ g(f(-x)) &= g(f(x)) \quad \text{also even} \end{aligned}$$

- d) Which of the following attributes are always unchanged by a vertical dilation?

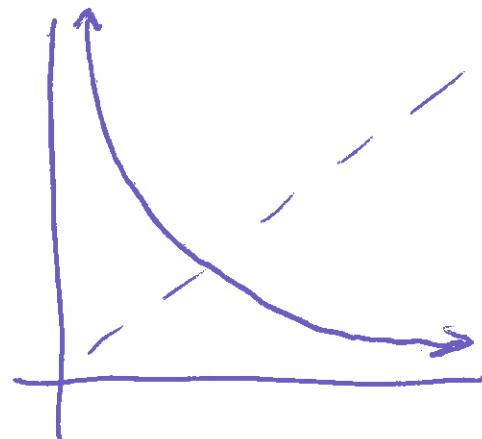
x-intcpts, y-intcpt, symmetry, domain, range, vertical asymptotes

- e) Same as previous question but for a reflection over the y-axis?

x-intcpts, y-intcpt, symmetry, domain, range, vertical asymptotes

8. Sketch a function with domain $x > 0$ such that $f(x) > 0$ for all values of x , $f(x)$ is decreasing for all values of x , $f(x)$ is concave up for all values of x and $f(x) = f'(x)$. Or explain why such a function is impossible.

for example $y = \frac{1}{x}$, $x > 0$



9. Given the function $f(x)$ at right,

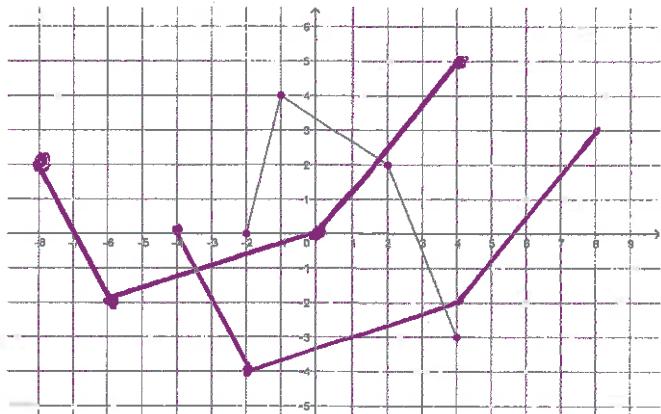
sketch $y = 2 - f\left(\frac{1}{2}x + 2\right)$

$$= 2 - f\left[\frac{1}{2}(x+4)\right]$$

$$T \langle -4, 2 \rangle$$

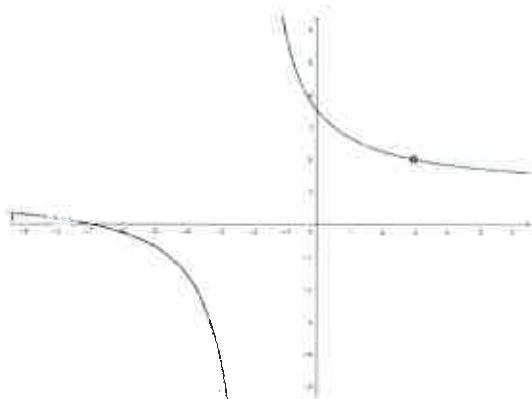
Do this
first
(in red)

$\begin{cases} \uparrow x\text{-axis} \\ \text{Horiz Stretch } x2 \end{cases}$



10. Write a function that corresponds to the graph at right.

- Hole @ $x=3 \rightarrow \frac{x-3}{x-3}$
- vert asympt @ $x = -2 \rightarrow \frac{1}{x+2}$
- x -intcpt @ $x=-7 \rightarrow \frac{x+7}{1}$



Try: $y = \frac{(x-3)(x+7)}{(x-3)(x+2)}$

Check: $\lim_{x \rightarrow 3} = \frac{0}{0}$, factor $\lim_{x \rightarrow 3} \frac{x+7}{x+2} = \frac{10}{5} = 2 \quad (3, 2) \checkmark$

Check: $\lim_{x \rightarrow \infty} = \frac{\infty}{\infty}$, obliterate: $\frac{x^2}{x^2} = 1$

horiz asympt $y = 1 \quad \checkmark$