Precalc BC Higher Dimensions – REVIEW Name: Key

1.

a) Find the plane containing S(-1, 2, 2); T(-2, 1, 0); U(0, 1, 1)

$$\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \qquad x + 3y - 2z = 1$$

b) Find the line containing A(3, 4, 5) and B (-1, 0, 1)

$$(3-4t,\ 4-4t,\ 5-4t)$$

c) Find the intersection of the plane and the line.

$$(3-4t) + 3(4-4t) - 2(5-4t) = 1 \rightarrow t = \tfrac{1}{2} \rightarrow (1, 2, 3)$$

2. Consider the three points P(-1, 0, 4); Q(5, 3, 7); R(-5, -2, 2)

a) Show that the three points are collinear by finding the equation of the line $\overleftrightarrow{PQ}$ and showing that R is on this line.

$$\overleftrightarrow{PQ} = (-1+6t,\ 3t,\ 4+3t), \text{ for } t = \tfrac{-2}{3} \rightarrow R(-5, -2, 2)$$

b) Show that the three points are collinear by finding the measure of $\angle QPR$ using the dot product.

$$\overrightarrow{PQ} = \langle 6, 3, 3 \rangle \qquad \cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{-36}{\sqrt{54} \cdot \sqrt{24}} = -1$$

$$\overrightarrow{PR} = \langle -4, -2, -2 \rangle \qquad \theta = 180^\circ$$

c) Write the equation of the sphere with diameter $\overline{PR}$.

$$\text{mdpt } \overline{PR} = (-3, -1, 3) \in M, \qquad MR = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$(x+3)^2 + (y+1)^2 + (z-3)^2 = 6$$

d) Find the intersection of the line $\overleftrightarrow{PQ}$ with the plane: x + 3y - 2z = 6

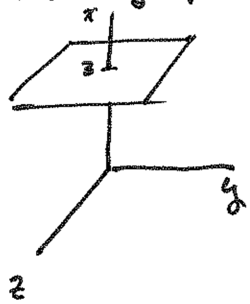
$$(-1+6t) + 3(3t) - 2(4+3t) = 6$$

$$\rightarrow t = \tfrac{5}{3} \rightarrow (9, 5, 9)$$

3. Identify by name and sketch. Be sure to indicate axes and intercepts.

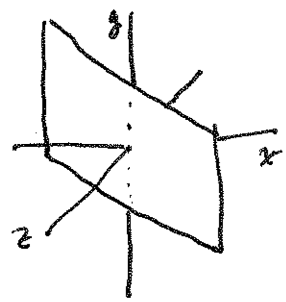
a) $x = 3$

Plane $\parallel$ to yz-plane



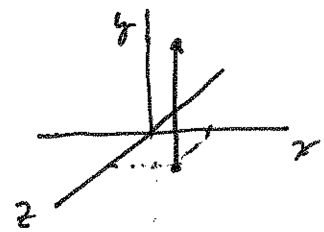
b) $x = z$

Diagonal plane thru y-axis



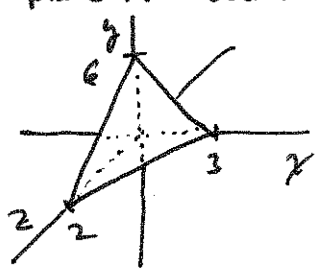
c) $(1, t, 1)$

line $\parallel$ to y-axis



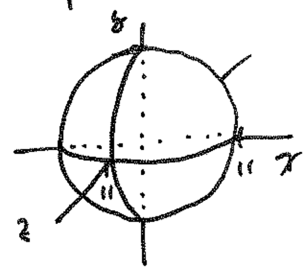
d) $4x + 2y + 6z = 12$

plane in 1st octant



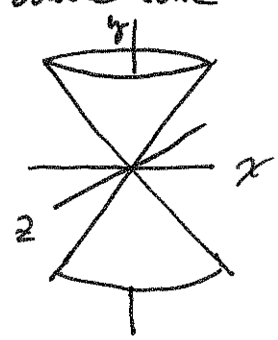
e) $x^2 + y^2 + z^2 = 121$

sphere



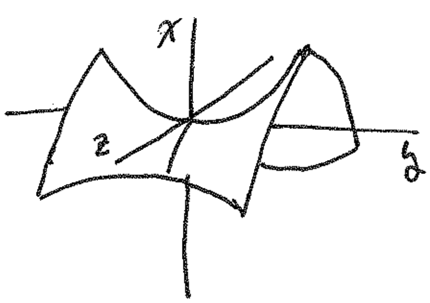
f) $x^2 + z^2 = y^2$

double cone



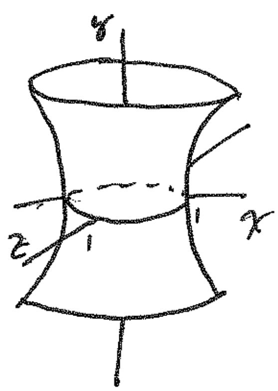
g) $x = y^2 - z^2$

Hyperbolic Paraboloid



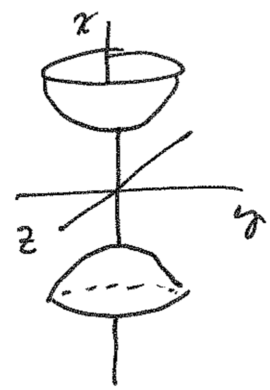
h) $x^2 - y^2 + z^2 = 1$

Hyperboloid of 1 sheet



i) $x^2 - y^2 - z^2 = 1$

Hyperboloid of 2-sheets



4. Find the endpoint P of segment $\overline{QP}$ with Q(7, -2, 4) and midpoint M(4, 3, 0).

$$\frac{x+4}{2} = 4, \quad \frac{y-2}{2} = 3, \quad \frac{z+4}{2} = 0 \quad \Rightarrow (1, 8, -4)$$

7. The point P(7, -2, 5) is first rotated 90° around the *x*-axis and then reflected through *yz*-plane. Where does it end up?

$P(7,2,5) \xrightarrow{R_{90\ x\text{-axis}}} P'(7,-5,-2) \xrightarrow{\Gamma_{yz}} P''(-7,-5,-2)$

8. Describe each of these as a single transformation:

a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\Gamma_{(0,0,0)}$ $\quad$ $R_{90°\ x\text{-axis}}$ $\quad$ $\Gamma_{xy\text{-plane}}$

9. Consider the **4-D** points P(1, 2, 2, 3); Q(2, 0, -1, 3) and R(3, 1, 1, 4)

a) Find $|\overline{QP}|$ and $|\overline{QR}|$

$\vec{QP} = \langle -1, 2, 3, 0 \rangle$ $\quad$ $\vec{QR} = \langle 1, 1, 2, 1 \rangle$

$|QP| = \sqrt{1^2+2^2+3^2+0^2} = \sqrt{14}$ $\quad$ $|\vec{QR}| = \sqrt{1^2+1^2+2^2+1^2} = \sqrt{7}$

b) Find the measure of $\angle QPR$ using dot product.

$$\cos\theta = \frac{\vec{QP}\cdot\vec{QR}}{|QP||QR|} = \frac{-1+2+6+0}{\sqrt{14}\cdot\sqrt{7}} = \frac{7}{7\sqrt{2}} = \frac{1}{\sqrt{2}} \rightarrow \theta = 45°$$

c) Find the exact area of ΔQPR (use your answer from a).

$$\frac{1}{2}(a)(b)\sin\theta$$
$$= \frac{1}{2}(\sqrt{14})(\sqrt{7})\sin 45° = \frac{7\sqrt{2}}{2\sqrt{2}} = \frac{7}{2}$$

d) Write an equation of the line through P, parallel to $\overline{QR}$.

$(1+t,\ 2+t,\ 2+2t,\ 3+t)$

5. Write the equation of...

a) A plane parallel to the *xy*-plane with z intercept (0,0,4).

$z = 4$

b) A plane parallel to 2x - 3y + z = 1, and containing the point (6,6,4).

$2(6) - 3(6) + (4) = -2$

$\rightarrow 2x - 3y + z = -2$

c) A plane with intercepts (6, 0, 0), (0, 4, 0), and (0, 0, 8).

$\frac{x}{6} + \frac{y}{4} + \frac{z}{8} = 1$

d) What is the volume of the pyramid bounded by the three coordinate planes and the plane in part (c)?

$\frac{1}{3!}(6)(4)(8) = 32$

6. A submarine on the surface of the ocean with a velocity of 28 knots/hour and a bearing of 157° begins a dive with an angle of descent of 22°. A current of 7 knots/hour flows parallel to the ocean floor with a bearing of 47°. Find a vector in rectangular form that describes the motion of the submarine relative to the ocean floor: $\langle N, E, Up \rangle$.

sub $\langle (28\cos 22°)(\sin 293°), (28\cos 22°)(\cos 293°), 28\sin 22° \rangle = \langle -23.9, 10.1, -10.5 \rangle$

current $\langle 7\sin 43°, 7\cos 43°, 0 \rangle = \langle 4.8, 5.1, 0 \rangle$

$\langle -19.1, 15.2, -10.5 \rangle$