Key

## Pre Calc BC CONICS -- Review

1. Given the parabola with equation $4x + y^2 = 0$.

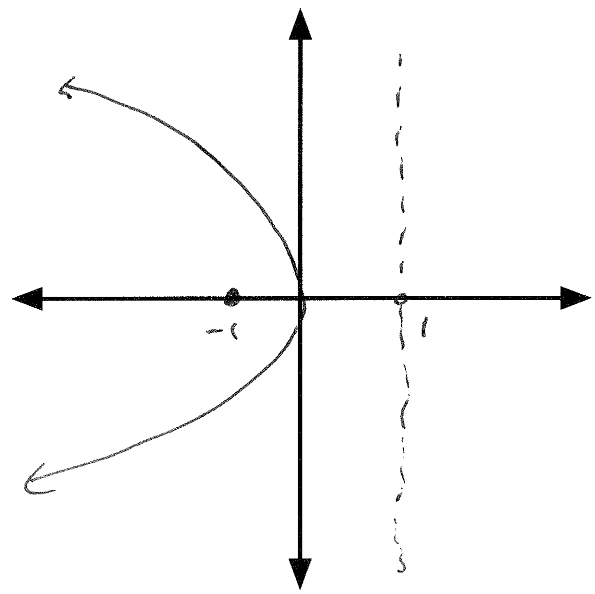
   a. Find the coordinates of the focus and the equation of the directrix. Then make a nice sketch.

   $x = -\frac{1}{4}y^2$

   $p = 1$

   focus $(-1, 0)$

   directrix $x = 1$




   b. Use the general substitution: $x' = x\cos\alpha - y\sin\alpha$ and $y' = x\sin\alpha + y\cos\alpha$ to get the equation for a *general* rotation of $\alpha$ of the above parabola. (ie. replace *x* and *y* in the original equation with *x'* and *y'*. Then simplify).

   $4(x\cos\alpha - y\sin\alpha) + (y\cos\alpha + x\sin\alpha)^2 = 0$

   $(\sin\alpha)^2 x^2 + (\sin 2\alpha)xy + (\cos\alpha)^2 y^2 + (4\cos\alpha)x - (4\sin\alpha)y = 0$

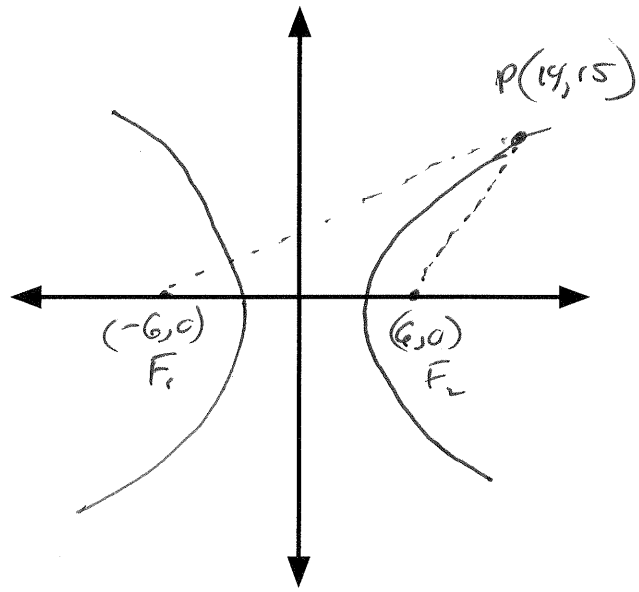
   c. Find the specific equation of the parabola for each value of $\alpha$ below (ie. plug in the indicated value for $\alpha$ and simplify).

   (1) 90° $x^2 - 4y = 0$

   (2) 45° $\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + 2\sqrt{2}x - 2\sqrt{2}y = 0$

   (3) 30° $\frac{1}{4}x^2 + \frac{\sqrt{3}}{2}xy + \frac{3}{4}y^2 + 2\sqrt{3}x - 2y = 0$

2. A hyperbola, centered at the origin, has a focus at (6, 0) and contains the point (14, 15). MAKE A SKETCH!




a. Find the second focus.

(-6, 0)

b. Calculate the difference of the focal radii.

$PF_1 = \sqrt{20^2+15^2} = 25$

$PF_2 = \sqrt{8^2+15^2} = 17$

$DFR = 8$

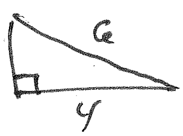
c. Find the eccentricity of the hyperbola.

$DFR = 8 \rightarrow a = 4$ (transverse radius)

$c = 6$ (focal dist)

$e = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

d. Find the equations of the asymptotes.




$b = \sqrt{36-16} = 2\sqrt{5}$

$y = \pm \frac{b}{a} x \rightarrow y = \pm \frac{\sqrt{5}}{2} x$

e. Write the equation of the hyperbola

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

f. Suppose the $x^2$ and $y^2$ terms are exchanged in your equation above (and their respective denominators as well). Revise your answers to parts b, c and d.

b) $dfr = 2b = 4\sqrt{5}$

c) $e = \frac{c}{b} = \frac{6}{2\sqrt{5}} = \frac{3}{\sqrt{5}}$

d) same

3. Given the conic: $4x^2 - 11xy + 9y^2 - 36 = 0$

a) Identify

$B^2 - 4AC = 11^2 - 4(4)(9)$

$= -23 \rightarrow$ ellipse

b) Determine angle of rotation

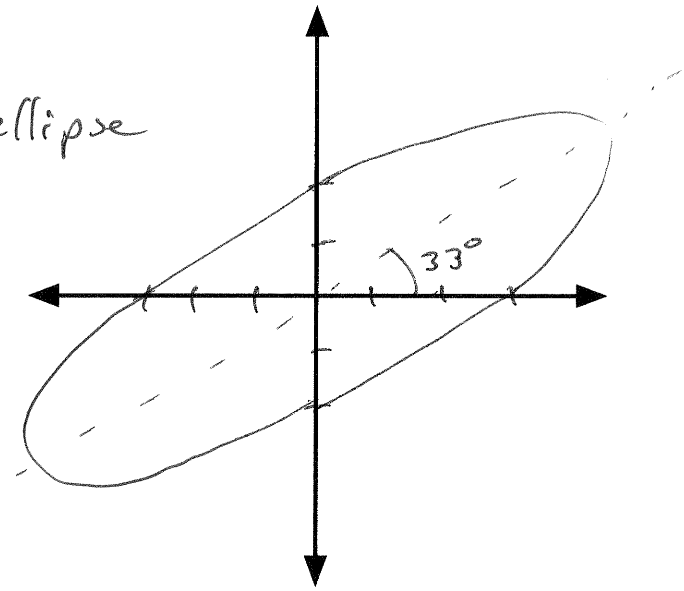
$\tan 2\theta = \frac{B}{A-C} = \frac{-11}{-5}$

$\theta \approx 33^\circ$

c) Find the intercepts and make a sketch. Then check on geogebra.

$y=0 \rightarrow x = \pm 3$

$x=0 \rightarrow y = \pm 2$




4. There is a unique conic passing through the five points A(-2, 1), B(-1, 3), C(2, 2), D(4, 3) and E(2, -1).

   a. Use matrices for to find the equation. (*Make your 5 x 1 matrix = a column of 25's to make life easier).*

   $8x^2 - 15xy - 12y^2 + 21x + 42y = 50$

   b. Identify the conic.

   $B^2 - 4AC = (-15)^2 - 4(8)(-12) = 609$

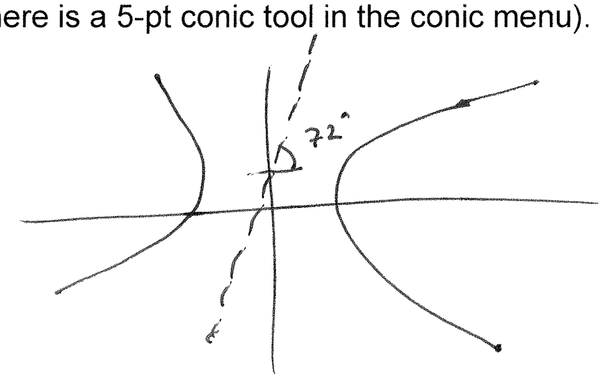
   Hyperbola

   c. Determine the angle of rotation.

   $\tan 2\theta = \frac{B}{A-C} = \frac{-15}{8 - -12} = \frac{-15}{20} = \frac{-3}{4}$

   $\theta \approx 72^\circ$

   d. Verify on geogebra (there is a 5-pt conic tool in the conic menu).



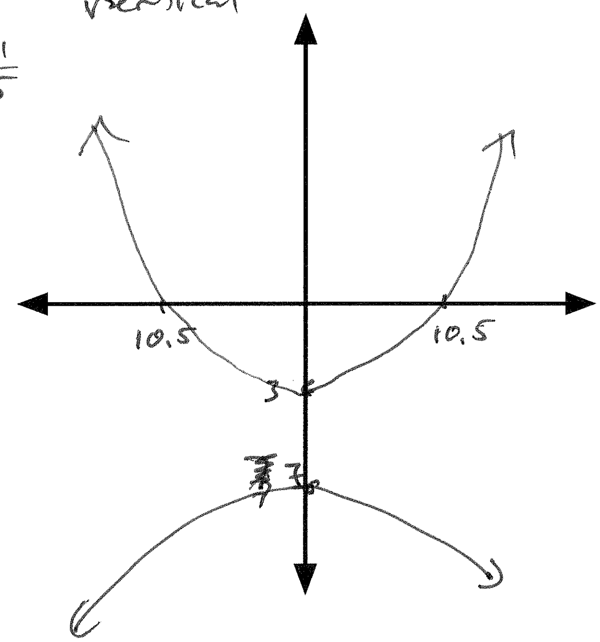
5. Given the conic described by $r = \dfrac{21}{2-5\sin\theta}$.

a. Describe and the conic (indicate type, attributes and eccentricity).

$r = \dfrac{\frac{21}{2}}{1-\frac{5}{2}\sin\theta}$ $e = \frac{5}{2}$ vertical

hyperbola $k = \frac{-21}{5}$

b. Sketch.




c. Write an equation in *rectangular* form for this conic.

center $(0,-5)$

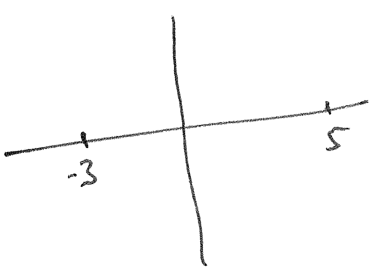
transv rad $= 2$

focal dist $= 5$

transv rad $= \sqrt{5^2-2^2} = \sqrt{21}$

$$\frac{(y+5)^2}{4} - \frac{x^2}{21} = \cancel{1}$$

d. Write an equation in polar form for the ellipse with focus at the origin and vertices at (-3, 0) and (5, 0).




center $(1,0)$

maj axis 8

~~min~~ focal dist $= 3$

$$\frac{(x-1)^2}{16} + \frac{y^2}{7} = 1$$


$b = \sqrt{16-9} = \sqrt{7}$

6. **Circle the correct answer.** A double cone is intersected by a plane parallel to the axis of the cone. The resulting intersection is a conic whose eccentricity is:

| | | |
|---|---|---|
| a) 0 | b) between 0 and 1 | c) less than zero |
| d) 1 | e) greater than 1 | f) unknowable |

**Extra Conics - Review** Name Key

1. Given $x^2 + xy = 1$

a) Identify the type of conic.

$B^2 - 4AC = 1 - 4(1)(0) = 1$

Hyperbola

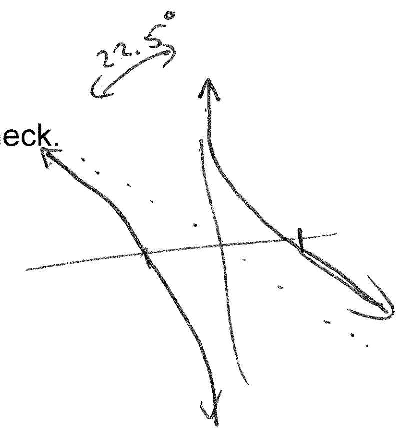
b) Find the angle of rotation.

$\tan 2\theta = \frac{B}{A-C} = \frac{1}{1} = 1 \qquad \theta = 22.5^\circ$

c) Find the *x* and *y* intercepts and make a sketch.

$x = 0 \rightarrow y =$ impossible!

$y = 0 \rightarrow x = \pm 1$




d) Solve for y and graph in your calculator to check.

$y = \frac{1-x^2}{x}$

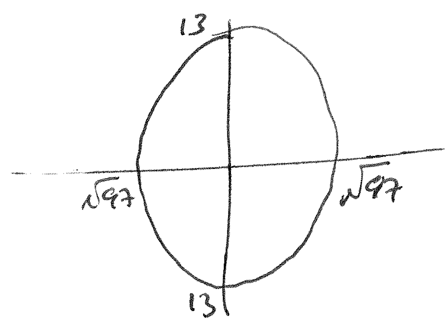
2. Sketch and find eccentricity.

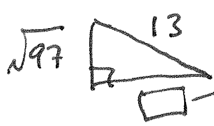
a) $169x^2 + 1014x + 97y^2 - 776y - 13,320 = 0$

$169(x^2 + 6x + 9) + 97(y^2 - 8y + 16) = 13,320 + 1521 + 1552$

$169(x+3)^2 + 97(y-4)^2 = 16,393$

$\frac{(x+3)^2}{97} + \frac{(y-4)^2}{169} = 1$

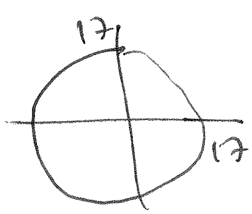



$\rightarrow \sqrt{72} = 6\sqrt{2}$

$e = \frac{c}{b} = \frac{6\sqrt{2}}{13}$

b) $\frac{x^2}{289} + \frac{y^2}{289} = 1$



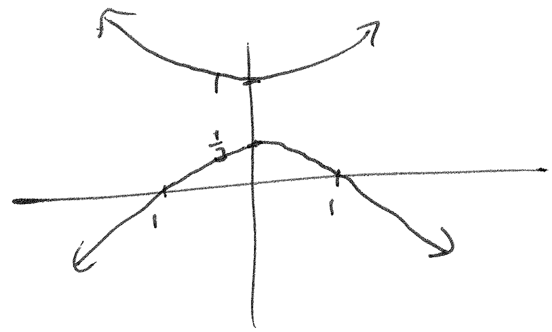

circle $\rightarrow e = 0$

3. Given $r = \dfrac{1}{1+2\sin\theta}$

a) complete the table:

b) Make a sketch.

| $\theta$ | 0 | π/2 | π | 3π/2 |
|---|---|---|---|---|
| r | 1 | 1/3 | 1 | −1 |

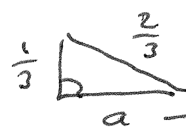



c) Find a, b, c, e

$b = \dfrac{1 - \frac{1}{3}}{2} = \dfrac{1}{3}$

$e = \dfrac{c}{b} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$

$c = \dfrac{1 + \frac{1}{3}}{2} = \dfrac{2}{3}$




$\rightarrow \dfrac{1}{\sqrt{3}}$

d) Write an equation in rectangular form.

$\cancel{\dfrac{y^2}{\frac{1}{9}}}$

$$\frac{\left(y - \frac{2}{3}\right)^2}{\frac{1}{9}} - \frac{x^2}{\frac{1}{3}} = 1$$

e) Rotate the conic (in polar form) by π/3.

$$r = \frac{1}{1+2\sin\left(\theta - \frac{\pi}{3}\right)}$$