## Pre Calc BC

Isomorphisms
Name $\qquad$

Examine the following pairs of groups and determine whether they are isomorphic. Make a list of elements and their orders to get started.

1. and $\left(\mathbb{Z}_{4}, \oplus\right)$.

| $*$ | $\mathbf{w}$ | $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{w}$ | z | y | w | x |
| $\mathbf{x}$ | y | z | x | w |
| $\mathbf{y}$ | w | x | y | z |
| $\mathbf{z}$ | x | w | z | y |

2. The rotational symmetry group of a square, and $\left(\{2,4,6,8\}, \otimes_{10}\right)$ (What is the identity in the second group?).
3. The permutation group of 2 elements, and $\left(\mathbb{Z}_{2}, \oplus\right)$.
4. $\left(\mathbb{Z}_{5}^{*}, \otimes\right)$, and $(\{1,3,7,9\}, \varnothing)$, where $x \varnothing y=$ the last digit in the product $x y$.
5. $\left(\mathbb{Z}_{5}, \oplus\right)$ and the rotational symmetry group for a regular pentagon.
$6 .\left(\{1,2,4,5,7,8\}, \otimes_{9}\right)$ and $\left(\left\{1,-1,\left(\frac{1}{2}+\frac{\sqrt{3}}{2} i\right),\left(-\frac{1}{2}+\frac{\sqrt{3}}{2} i\right),\left(-\frac{1}{2}-\frac{\sqrt{3}}{2} i\right),\left(\frac{1}{2}-\frac{\sqrt{3}}{2} i\right)\right\}, \times\right)$.
6. Can you draw a figure whose symmetry group is isomorphic to the group shown at right? (Can you draw a figure for which this represents the entire symmetry group, or is this merely a sub-group?)

| $*$ | C | I | P | S |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{C}$ | S | P | I | C |
| I | P | S | C | I |
| P | I | C | S | P |
| S | C | I | P | S |

