

## Pre Calc BC      Matrices and Groups Review

1. Which of the five groups below are isomorphic?

$$G = (\mathbb{Z}_5^*, \otimes); \quad H = (\mathbb{Z}_4, \oplus); \quad J = \text{square rotation group}$$

$$K = (\{1, -1, i, -i\}, \times) \quad M = \text{rectangle symmetry group}$$

- a)  $G \sim H$  only      b)  $H \sim J$  only      c)  $K \sim M$  only  
 d)  $H \sim J \sim K$  only      e)  $G \sim H \sim J \sim K$

2. Let  $a$  be an element of group  $G$ , a group of order 15. Which of the statements below must not be true?

- a)  $a$  is the identity      b)  $a$  is its own inverse (but not the identity)  
 c)  $a^4 = a$       d)  $a$  is in a subgroup of  $G$   
 e)  $a$  is not in a subgroup of  $G$

3. Consider the operator  $*$  such that  $a * b = 2ab$ . Which of these is false?

- a) there is an identity for  $*$       b)  $*$  is associative  
 c)  $*$  is commutative  
 d)  $*$  is closed for even numbers      e)  $*$  forms a group with the even numbers

4. Which of these is a cyclic group?

- a)  $(\mathbb{Z}_7^*, \otimes)$   
 b) the symmetry group for a square  
 c) the symmetry group for an equilateral  $\Delta$   
 d) the permutation group of four elements  
 e) the permutation group of 5 elements

5. Given  $\begin{bmatrix} 2 & 3 & a \\ b & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \end{bmatrix}$

the sum of  $a$  and  $b$  is

- a) 1    b) -1    c) 4    d) -3    e) -4

6. The transformation matrix  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$  represents

- a)  $r_{y=x}$       b)  $r_{y=-x}$       c)  $R_{180^\circ}$   
 d)  $R_{90^\circ}$       e)  $R_{-90^\circ}$

7. Given the system of equations:

$$\begin{cases} x - 3y + z = -2 \\ 2x + 3y - 4z = -4 \\ x + y - z = 0 \end{cases}$$

the product  $xyz$  equals

- a) 0    b) 6    c) -3    d) 10    e) -12

8. If  $k = \frac{453!}{450!3!}$ , then

- a)  $k > 10^{100}$       b)  $10^{10} \leq k < 10^{100}$   
 c)  $10^5 \leq k < 10^{10}$       d)  $10 \leq k < 10^5$   
 e)  $k < 10$

9.  $(p, q)$  is called a *lattice point* if  $p$  and  $q$  are both integers. How many lattice points lie in the area between the two curves  $x^2 + y^2 = 9$  and  $x^2 + y^2 - 6x + 5 = 0$ ?

- a) 0    b) 1    c) 2    d) 3    e) 4

10. If the determinant of the matrix  $\begin{bmatrix} 7 & a \\ 4 & 3 \end{bmatrix} = 1$

then  $a$  must equal

- a) -1    b) 0    c) 1    d) 2    e) 5

11. If the matrix  $\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$  is in a multiplicative group, which of these must also be in the group?

I.  $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$       II.  $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$       III.  $\begin{bmatrix} 0 & 0.5 \\ -1 & 0 \end{bmatrix}$

- a) I only      b) II only      c) III only  
 d) I and II      e) II and III