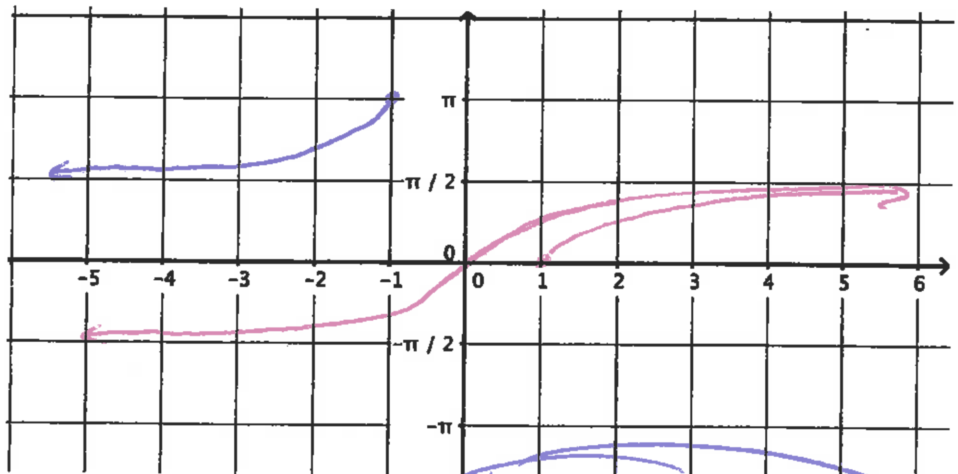
PreCalc BC Trig. Review # 2 Name Steve

* means **NO CALCULATOR**

1. *Use a sketch to determine the value of $\lim_{x\to\infty} \sec^{-1}(x) - \lim_{x\to\infty} \tan^{-1}(x)$



$\frac{\pi}{2} - \frac{\pi}{2} = 0$

2. Determine which of the functions below can be expressed as a single sine function. If it is possible, do it!

a) $g(x) = \cos^4 x - \sin^4 x$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= (1)(\cos 2x)$

$= \sin\left(\frac{\pi}{2} - 2x\right)$

b) $h(x) = \sin(x) + \sin(2x)$

Not a sinusoid! (different periods)

c) $f(x) = 3\sin x - 5\cos x$

(use calculator)

$= 5.83 \sin(x - 1.03)$

3. *Find all solutions in the interval [0, 2π), **ALGEBRAICALLY** :

a) $\sin^2(x) = \cos^2(x) - \frac{1}{\sqrt{2}}$

$\cos^2 x - \sin^2 x = \frac{1}{\sqrt{2}}$

$\cos 2x = \frac{1}{\sqrt{2}}$

$0 \le 2x < 4\pi$

$2x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$x = \left\{\frac{\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\right\}$

b) $2\tan(x) = \sqrt{3}\left(1 - \tan^2(x)\right)$

$\frac{2\tan x}{1 - \tan^2 x} = \sqrt{3}$

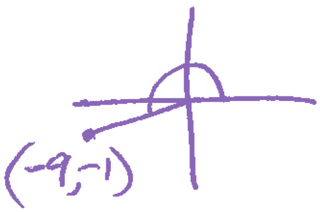
$\tan 2x = \sqrt{3}$

$0 \le 2x < 4\pi$

$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$

$x = \left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}$

4. Find the measure of the angle formed by terminal point (-9, -1) and the positive x-axis. Assume ($0 \le \theta < 2\pi$).

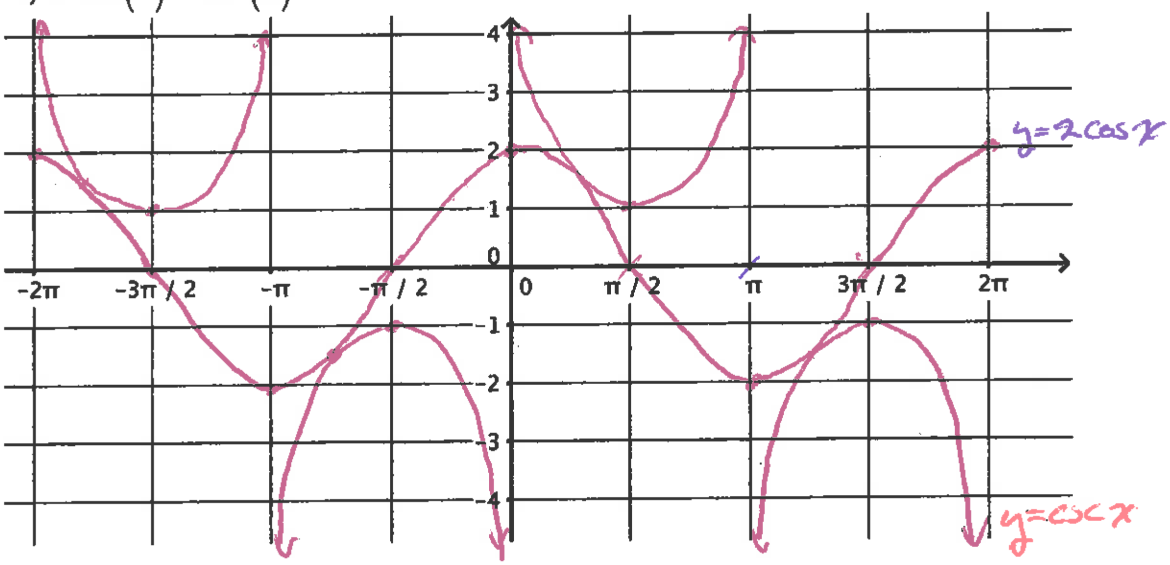


$$\theta = \pi + \tan^{-1}\left(\frac{1}{9}\right)$$

$$= \underline{\underline{3.25}}$$

5. *Investigate the following *graphically*. Make a good sketch and conjecture.

a) $2\cos(x) = \csc(x)$



b) Solve the following *algebraically*: $2\cos(x) = \csc(x)$ for all values of x in the interval $[-2\pi, 2\pi]$. Compare your answer to part *a*.

$$2\cos x = \csc x$$

$$2\sin x \cos x = 1$$

$$\sin 2x = 1$$

$$-4\pi \le 2x \le 4\pi$$

$$2x = -\frac{7\pi}{2}, \frac{-3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}$$

$$x = \left\{-\frac{7\pi}{4}, \frac{-3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$$

6. The center of a Ferris wheel is 45 feet high, and the radius of the wheel is 40 feet. It takes 96 seconds to complete a full revolution and the wheel turns counter-clockwise.

a) Write a sinusoid to model the height of a rider who starts at the 3:00 position. $a = 40,\ b = \frac{2\pi}{per} = \frac{2\pi}{96} = \frac{\pi}{48},\ c = 0,\ d = 45$

$$y = 40 \sin\left(\frac{\pi}{48} x\right) + 45$$

b) Use you equation to determine when the rider is first at a height of 65 feet.

$40 \sin\left(\frac{\pi}{48} x\right) + 45 = 65$

$\sin\left(\frac{\pi}{48} x\right) = \frac{1}{2}$

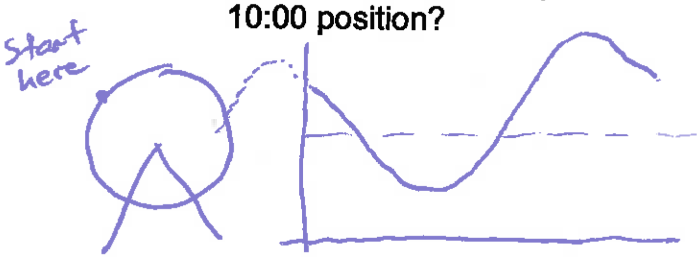
$\frac{\pi}{48} x = \frac{\pi}{6}$

$x = 8$ sec

c) Determine the linear velocity of the rider.

$V = \omega r = \left(\frac{2\pi}{96}\right)(40) = \frac{5\pi}{6} \approx 2.6$ ft/sec

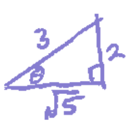
d) How would your equation from part *a* change if the rider started at the 10:00 position?



10:00 means translation to left by $\frac{5}{12}$ of rotation. $\left(\frac{5}{12}\right)(96) = 40$ sec

$$y = 40 \sin\left[\frac{\pi}{48}(x + 40)\right] + 45$$

7. *If $\sin\theta = \frac{2}{3}$, find each of the following



a) $\cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9}$

$= \frac{1}{9}$

b) $\cot 2\theta$

$= \frac{1}{\tan 2\theta}$

$= \frac{1 - \tan^2\theta}{2\tan\theta}$

$= \frac{1 - \left(\frac{2}{\sqrt{5}}\right)^2}{2\left(\frac{2}{\sqrt{5}}\right)}$

$= \frac{1 - 4/5}{4/\sqrt{5}}$

$= \frac{1}{5} \cdot \frac{\sqrt{5}}{4} = \frac{\sqrt{5}}{20}$

c) $\sin\frac{\theta}{2}$

$= \sqrt{\frac{1 - \cos\theta}{2}}$

$= \sqrt{\frac{1 - \frac{\sqrt{5}}{3}}{2}}$

$= \sqrt{\frac{3 - \sqrt{5}}{6}}$

d) $\cos\left(\theta - \frac{\pi}{2}\right)$

$= \cos\left(\frac{\pi}{2} - \theta\right)$

$= \sin\theta$

$= \frac{2}{3}$

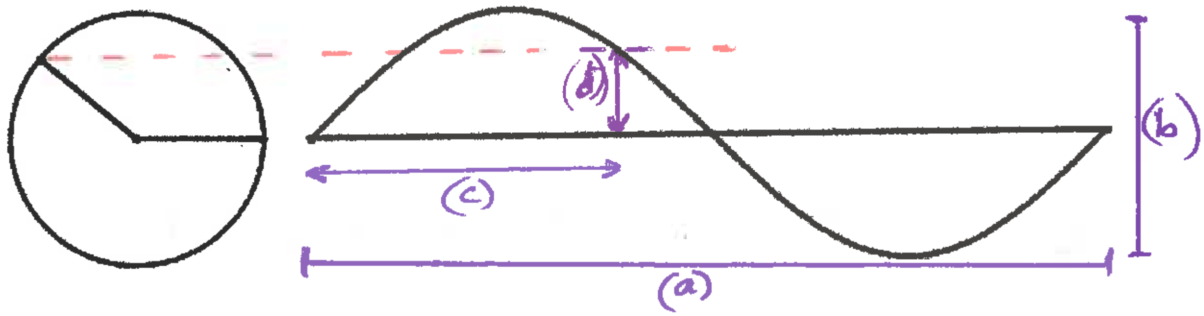
8. *Shown below is the graph of $\sin(\theta)$ for the unit circle shown. Indicate how each of the following is conveyed in the graph. *(ie. Identify and label a length corresponding to each attribute directly on the graph – use a highlighter!)*

a) The diameter of the circle.

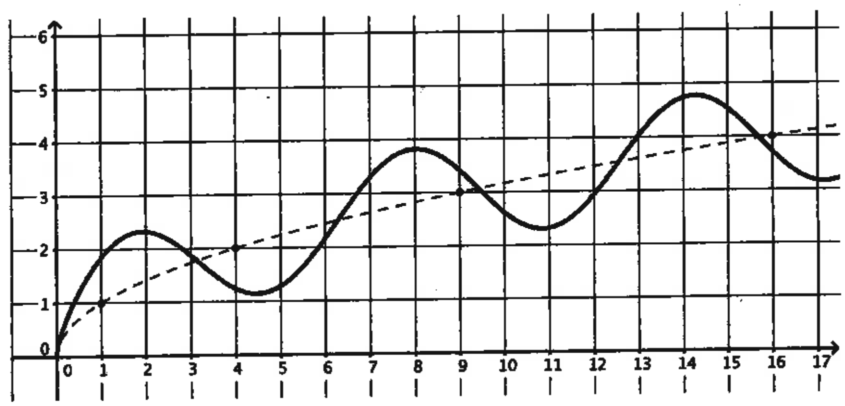
b) The circumference of the circle.

c) The arc length of θ

d) The $\sin(\theta)$.



9. Write an equation for the function shown below. *(Verify by typing your equation into Geogebra or Desmos).*



$y = \sin x + \sqrt{x}$

10. *Solve algebraically: $\sec^2(x) + \csc^2(x) + 3\sec(x) + 1 = \cot^2(x)$, $0 \le x < 2\pi$

$\boxed{\text{note: } \cot^2 x + 1 = \csc^2 x}$

$\sec^2 x + (\cot^2 x + 1) + 3\sec x + 1 = \cot^2 x$

$\sec^2 x + 3\sec x + 2 = 0$

$(\sec x + 1)(\sec x + 2) = 0$

$\sec x = -1$ or -2

$x = \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$